Worked Solutions

Edexcel C4 Paper K

1. (*a*) $V = \frac{4}{3}\pi r^3,\ \frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ **(1)**

(*b*) $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t}$

given $r = 10$ and $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.1,\ \frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi \times 10^2 \times 0.1$

$= 40\pi \ \text{cm}^3\ \text{s}^{-1}$ **(4)**

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2. (*a*) $2x + x \cdot \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} + \ln y + \frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{x}{y} + 1\right) = -(2x + \ln y)$

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{(2x + \ln y)}{\left(\frac{x}{y} + 1\right)}$

at $(3, 1)$ gradient $= -\left(\frac{6+0}{3+1}\right) = -\frac{3}{2}$ **(3)**

(*b*) $6x + 2x\frac{\mathrm{d}y}{\mathrm{d}x} + y \cdot 2 - 10y\frac{\mathrm{d}y}{\mathrm{d}x} + 16\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$\frac{\mathrm{d}y}{\mathrm{d}x}(2x - 10y + 16) = -(6x + 2y)$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $6x + 2y = 0$

or $y = -3x$

substitute $y = -3x$ into equation of curve,

$3x^2 + 2x(-3x) - 5(9x^2) + 16(-3x) = 0$

$-48x^2 - 48x = 0$

$-48x(x+1) = 0$

$x = 0$ or -1 **(5)**

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3. (*a*) at P $y = 0$.

$\cos t = 0$

$t = \frac{\pi}{2}$

at $t = \frac{\pi}{2},\ x = \frac{\pi^2}{4}$

coordinates of P are $\left(\frac{\pi^2}{4}, 0\right)$ **(2)**

(*b*) (i) area $= \int y\frac{\mathrm{d}x}{\mathrm{d}t}\ \mathrm{d}t = \int_0^{\frac{\pi}{2}} \cos t \cdot 2t\ \mathrm{d}t$ **(2)**

(ii) $A = \int_0^{\frac{\pi}{2}} 2t\frac{\mathrm{d}}{\mathrm{d}t}(\sin t)\ \mathrm{d}t$ (By parts)

$= \left[2t\sin t\right]_0^{\frac{\pi}{2}} - 2\int_0^{\frac{\pi}{2}} \sin t\ \mathrm{d}t$

$= \left[2t\sin t + 2\cos t\right]_0^{\frac{\pi}{2}}$

$= \pi + 0 - (0 + 2) = \pi - 2$ **(5)**

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4. (*a*) $f(x) = (1 - 9x^2)^{-\frac{1}{2}}$

$$= 1 + \left(-\frac{1}{2}\right)\left(-9x^2\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-9x^2\right)^2$$

$$= 1 + \frac{9}{2}x^2 + \frac{243}{8}x^4 \qquad \textbf{(3)}$$

(*b*) valid for $|9x^2| < 1$

$$|x^2| < \frac{1}{9}, |x| < \frac{1}{3} \qquad \textbf{(1)}$$

(*c*) (i) $\frac{(1+3x)^{\frac{1}{2}}}{(1-3x)^{\frac{1}{2}}} \times \frac{(1+3x)^{\frac{1}{2}}}{(1+3x)^{\frac{1}{2}}} = \frac{1+3x}{\sqrt{(1-9x^2)}}$ **(2)**

(ii) $(1+3x)\left(1 + \frac{9}{2}x^2 + \frac{243}{8}x^4\right)$

$$= 1 + \frac{9}{2}x^2 + \frac{243}{8}x^4 + 3x + \frac{27}{2}x^3 + \frac{729}{8}x^5$$

$$= 1 + 3x + \frac{9}{2}x^2 + \frac{27}{2}x^3 + \frac{243}{8}x^4 + \frac{729}{8}x^5 \qquad \textbf{(2)}$$

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5. (*a*) $A = 160, B = 50$

[N doubles as t increases by 10] **(3)**

(*b*) (i) $t = 10, m = 500\,e^{-1} = 184$ **(1)**

(ii) $300 = 500\,e^{-0.1t}$

$$\ln\frac{3}{5} = -0.1t$$

$$t = 5.1 \qquad \textbf{(3)}$$

(iii) $\frac{dm}{dt} = 500(-0.1)\,e^{-0.1t}$

when $t = 20$, $\frac{dm}{dt} = -6.77$ gram/year **(3)**

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6. (*a*) (i) $\int x \ln x \; dx = \int \ln x \frac{d}{dx}\left(\frac{x^2}{2}\right) dx$ (By parts)

$$= \frac{x^2}{2}\ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} \; dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + c \qquad \textbf{(3)}$$

(ii) $\int \ln x \; dx = \int \ln x \frac{d}{dx}(x) \; dx$

$$= x\ln x - \int \frac{1}{x} \cdot x \; dx$$

$$= x\ln x - x + c \qquad \textbf{(3)}$$

(*b*) let $I = \int_1^{-2} x\sqrt{x+3} \; dx$ let $u = x + 3$, $\frac{du}{dx} = 1$

$\therefore \quad I = \int_4^1 (u-3)u^{\frac{1}{2}} \; du$ when $x = -2$, $u = 1$; $x = 1$, $u = 4$

$$= \int_4^1 \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) \; du$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} - 2u^{\frac{3}{2}}\right]_4^1 = \frac{8}{5} \qquad \textbf{(6)}$$

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7. (a) $\frac{dV}{dt} = 10 - \frac{V}{4}$

$4\frac{dV}{dt} = 40 - V, -4\frac{dV}{dt} = V - 40$ **(3)**

(b) $\int \frac{1}{V-40}dV = -\frac{1}{4}\int dt$

$\ln(V-40) = -\frac{1}{4}t + c$

$V = 100, t = 0: \quad \ln 60 = 0 + c$

$\therefore \quad \ln(V-40) = -\frac{1}{4}t + \ln 60$

$\ln\frac{(V-40)}{60} = -\frac{1}{4}t$

$\frac{V-40}{60} = e^{-\frac{1}{4}t}$

$V = 60e^{-\frac{1}{4}t} + 40$ **(7)**

(c) as $t \to \infty \quad V \to 40.$ **(1)**

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8. (a) l and m are perpendicular $\Rightarrow \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ 2 \end{pmatrix} = 0$

$b - 4 = 0$

$b = 4$

l and m intersect $\Rightarrow 2 = 1 + \mu a$ ... [A]

$-1 + \lambda = -2 + \mu \times 4$ ... [B]

$3 - 2\lambda = -5 + \mu \times 2$ ... [C]

solving [B] and [C], $\lambda = 3, \mu = 1$

substitute in [A], $a = 1$ **(6)**

(b) point of intersection is $\begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix}$ **(3)**

(c) $\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \sqrt{5}\sqrt{9}\cos\theta$, where θ = angle between lines

$0 + 2 - 4 = 3\sqrt{5}\cos\theta$

$\theta = 73°$ **(3)**
